

# Double Ridged Waveguide Phase Shifters for Broadband Applications

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**Abstract**—This paper presents an electromagnetic mode matching analysis of dual ridged waveguide phase shifters. The waveguide is loaded with a high dielectric material between the ridges, and the troughs are filled with ferrite toroids. The present structure offers nearly twice the bandwidth and less variation in nonreciprocity versus frequency as compared to the conventional dual toroidal phase shifter. An optimum design that maximizes nonreciprocity has been found. Effects of the dielectric constant and the trough dimensions are also presented. A differential phase shift of more than  $100^\circ/\text{cm}$  has been predicted for the new phase shifter.

## I. INTRODUCTION

MODERN microwave antennas, radars, and communication systems are moving toward higher frequencies. Requirements for those systems call for reconfigurable phase shifting structures that can operate over broad bandwidth, and have low loss at millimeter-wave frequencies. Presently, there are two types of electronic phase shifters: waveguide ferrite phase shifters and semiconductor-diode phase shifters. At frequencies above 30 GHz, diode phase shifters suffer from relatively high loss and narrow bandwidth [1]. With the lack of reliable low noise millimeter-wave amplifiers, the high loss of diode phase shifters makes them unacceptable for low noise figure purposes.

Recently, a slot line dual toroidal phase shifter has been developed [2] to increase the bandwidth and nonreciprocity of ferrite phase shifters. However, slot lines have relatively high losses ( $250^\circ/\text{dB}$ ) as compared to waveguides (more than  $500^\circ/\text{dB}$ ). In addition, the slot-to-ground plane connection inside a waveguide is not easy to produce.

The present paper investigates a novel approach to increase the bandwidth of waveguide phase shifters [Fig. 1(a)] without sacrificing low loss or ease of construction. The new structure is comprised of two ferrite toroids inside a ridged waveguide. The ridges are separated by a high dielectric constant material, and the troughs are filled with two toroids as shown in Fig. 1(b). Since ridged waveguides are inherently very broadband (typically 3:1 bandwidth), the new phase shifter is expected to have a broadband of operation.

Manuscript received March 26, 1993; revised June 24, 1993.

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IEEE Log Number 9213011.

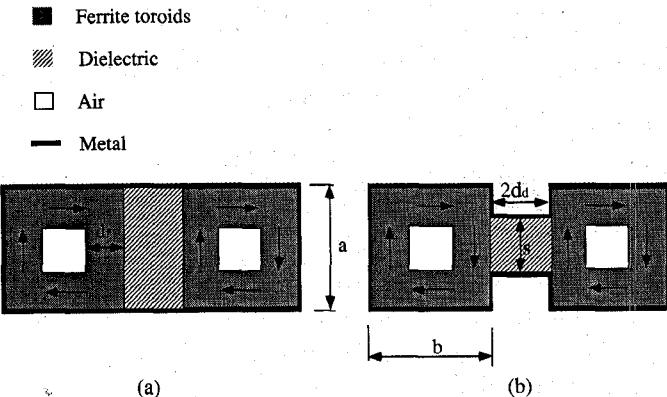


Fig. 1. Geometry of the dual toroidal phase shifter: (a) conventional, (b) ridged.

When a ferrite layer has two boundaries with air, the field ellipticities at the boundaries are opposite in sign [2]–[4]. Since the differential phase shift results from the interaction between these elliptic fields and the magnetic material [4], the opposite ellipticities counteract each other and result in a poor nonreciprocal behavior for single layer structures. Adding a layer of high dielectric materials at one of the ferrite boundaries replaces one of the aforementioned ellipticities by a co-acting ellipticity [2]–[4].

The present approach is to use two oppositely magnetized ferrite layers [see Fig. 1(b)] in addition to the high dielectric material. Although the two magnetic layers have opposite field ellipticities, their nonreciprocity adds because of the opposite magnetizations. Note that the high dielectric constant layer is used to prevent magnetic leakage from one ferrite layer to another, and to further enhance the nonreciprocal performance.

The analysis in the present work is based on the mode matching technique in the spectral domain [5]–[7]. Spectral domain techniques can easily be adopted for a wide variety of structures. However, the formulation of a Green's function represents a major complexity in that approach. Solving for a Green's function as a boundary value problem is quite difficult for magnetic substrates and becomes more complicated for multilayer structures. In addition, Green's functions in such a procedure will only be useful for the specific structures for which they are derived. A more flexible approach is to find the transmission matrix [3], [4] of the medium and then use this matrix to find the Green's function. The analysis is presented in

Section II and is used in Section III to calculate the propagation constant of the dual toroidal ridge waveguide phase shifter. The effects of the new structure on non-reciprocity and bandwidth are also presented in Section III, and the paper is concluded in Section IV.

## II. THEORY

### A. Green's Function Formulation

This section presents the Green's function used in the full-wave analysis of the multilayer ridge waveguide structures. The tangential ( $x$  and  $y$ ) components of the electric fields and surface currents are Fourier-transformed according to

$$\bar{F}(x, y) = \frac{1}{a} \int_{x=-\infty}^{\infty} e^{jk_x x} \sum_{i=-\infty}^{\infty} \tilde{F}(k_x, k_{yi}) e^{jk_{yi} y} dk_x \quad (1)$$

where the tilde ( $\sim$ ) denotes the Fourier transform. The conducting sidewalls restrict  $k_{yi}$  to the values  $\pi/a$ , where  $i$  is even, so the Fourier transform with respect to  $y$  is discrete. Note that  $k_{yi}$  is different for different cross sections of the waveguide. For the ridges,  $k'_{yi}$  takes the discrete values of  $i\pi/s$ , and  $s$  is the ridge separation. To find Green's function at the plane of the discontinuity between the ridges and the troughs, the surface current at the discontinuity, given by  $\tilde{J} = \hat{z} \times \tilde{H}$ , where  $\tilde{H}$  is the tangential magnetic field, is split into the superposition of two nonzero equivalent currents  $\tilde{J}$  and  $\tilde{J}'$ . This allows the definition of Green's functions of the trough and ridged regions.

$$\tilde{J} = \tilde{\bar{G}}_s(k_x, k_{yi}) \tilde{\bar{E}} \quad (2)$$

$$\tilde{J}' = \tilde{\bar{G}}'_s(k_x, k'_{yi}) \tilde{\bar{E}}' \quad (3)$$

Where, the ' denotes the ridged region. The above Green's functions are found in the spectral domain using the transmission matrices of the media. The transmission matrix  $\tilde{\bar{T}}(d)$  for a general dielectric/ferrite slab is a  $4 \times 4$  matrix given by [3]

$$\begin{bmatrix} \tilde{\bar{E}}_2 \\ \tilde{\bar{J}}_2 \end{bmatrix} = \tilde{\bar{T}}(d) \begin{bmatrix} \tilde{\bar{E}}_1 \\ \tilde{\bar{J}}_1 \end{bmatrix} = \begin{bmatrix} \tilde{\bar{T}}_E & \tilde{\bar{Z}}_T \\ \tilde{\bar{Y}}_T & \tilde{\bar{T}}_J \end{bmatrix} \begin{bmatrix} \tilde{\bar{E}}_1 \\ \tilde{\bar{J}}_1 \end{bmatrix} \quad (4)$$

where  $\tilde{\bar{T}}_E$ ,  $\tilde{\bar{Z}}_T$ ,  $\tilde{\bar{Y}}_T$ ,  $\tilde{\bar{T}}_J$  are  $2 \times 2$  submatrices of the transmission matrix  $\tilde{\bar{T}}$ .  $\tilde{\bar{E}}$  and  $\tilde{\bar{E}}_2$  are the transformed tangential electric fields at the boundaries of the layer.  $\tilde{\bar{J}}_1$  and  $\tilde{\bar{J}}_2$  are the transformed tangential components of the surface currents, just inside the  $n$ th surface of the layer. Note that the fields and currents in (4) are vector quantities of the form

$$\tilde{\bar{E}} = \begin{bmatrix} \tilde{\bar{E}}_x \\ \tilde{\bar{E}}_y \end{bmatrix}. \quad (5)$$

The Green's functions take into account the boundary conditions of each region. The even symmetry of the fields in the double ridged waveguide structure allows the split-

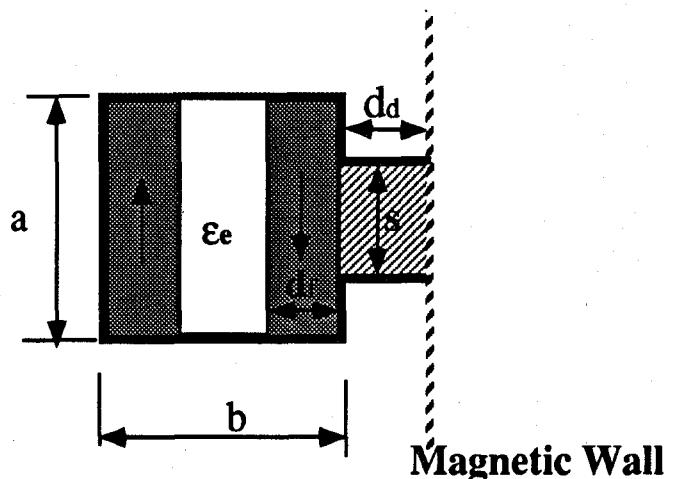


Fig. 2. Equivalent model of double ridged waveguide phase shifters.

ting of the guide by a magnetic wall at the center of the structure as depicted in Fig. 2. As shown, the ferrite toroids are modeled by retaining the vertical walls and replacing the horizontal walls by a dielectric layer of an effective dielectric constant  $\epsilon_e$  [3], [8].

Considering the simple case of a dielectric slab with either an electric wall or a magnetic wall at the back of the slab, the above Green's functions can be written as [3]

$$\tilde{\bar{G}}_s = \tilde{\bar{T}}_J \tilde{\bar{Z}}_T^{-1} \quad (6)$$

for the electric wall case, and

$$\tilde{\bar{G}}'_s = \tilde{\bar{Y}}_T \tilde{\bar{T}}_E^{-1} \quad (7)$$

for the magnetic wall case.

For multilayer dielectric/ferrite structures, the transmission matrices of individual layers are multiplied together in the correct sequence to find the total transmission matrix for the combined layers. Then, the above equations are used to find Green's functions of the multilayer structure.

### B. Full-Wave Formulation

The propagation constant of an infinitely long-ridged waveguide structure is found using the mode matching technique in the spectral domain [5]–[7]. Green's functions, as described in the previous section, are used to find the surface currents at the plane of discontinuity  $\bar{J}(y)$  in terms of the transformed electric fields  $\tilde{\bar{E}}$  using

$$\begin{aligned} \bar{J}(y) = & \frac{1}{a} \sum_{i=-\infty}^{\infty} \tilde{\bar{G}}_s(-\beta, k_{yi}) \tilde{\bar{E}}_i(k_{yi}) e^{jk_{yi} y} \\ & + \frac{1}{s} \sum_{i=-\infty}^{\infty} \tilde{\bar{G}}'_s(-\beta, k'_{yi}) \tilde{\bar{E}}'_i(k'_{yi}) e^{jk'_{yi} y}. \end{aligned} \quad (8)$$

The boundary conditions,  $\bar{J}(y) = 0$ , are enforced by multiplying the above equation by the field modal expansion functions ( $e^{jk_{yi} y}$ ) for different field regions (the ridged and the trough regions).

Application of the above procedure results in an admitt-

tance matrix; the solution for the propagation constant  $\beta$  is the value that forces the determinant of this admittance matrix to be zero. The difference between the propagation constants of the forward and reverse waves is used to find the nonreciprocal phase shift per unit length, as  $\Delta\phi = \beta_f - \beta_r$ . Three to five modal functions in each direction (total of 14 modal functions including the ridged and the trough regions) have been found to be sufficient for convergence.

### III. RESULTS

As a verification of the present theory, the computed differential phase for a conventional dual toroidal phase shifter (no ridges) is compared to the results given in [8] and [9]. The agreement was within 5% of the results in [8] and within 3% of the results in [9]. Fig. 3 shows the differential phase of a ridged waveguide phase shifter as compared to a conventional dual toroidal phase shifter. For the conventional waveguide phase shifter (with  $s = a$ ), the phase shift varies significantly versus frequency. The effect of double ridged waveguide is to flatten the differential phase over the bandwidth. This can simplify the control circuitry of the phase shifter.

The bandwidth of the present phase shifter is limited at the upper end by the excitation of higher order modes, and at the lower end by magnetic losses of the ferrite material [8]. For the ferrite material simulated in this paper, the low frequency was about 6 GHz. Fig. 4 shows the normalized propagation constant for ridged waveguide and conventional phasers. For single mode of operation, only one solution for  $\beta$  should exist for the same direction of propagation. Overmoding, where more than one solution exist, starts to occur in conventional waveguide phase shifters approximately at 9.5 GHz compared to 16.0 GHz for the ridged waveguide of the same outer dimensions. Thus, for  $s/a = 0.25$ , the bandwidth of the double ridged phase shifter (6–16 GHz) is about three times the bandwidth of conventional waveguide phase shifters (6–9.5 GHz). Fig. 5(a) shows the increase in the upper frequency limit, versus the ratio  $s/a$  of the ridged phase shifter normalized to that of conventional phase shifters. According to this figure, as the ridge separation increases, overmoding occurs at lower frequency, with the minimum occurring when the ridge height equals the sidewall separation ( $s/a = 1$ ); i.e., no ridges exist in the structure. Fig. 5(b) shows the corresponding differential phase per unit length at 6 GHz. The figure shows that as  $s/a$  increases, the nonreciprocity increases and reaches a maximum roughly at  $s/a = 0.5$ . This ratio has been found to change slightly versus frequency and dimensions of the waveguide. At this maximum, the nonreciprocity of the ridged waveguide phase shifter is about 14% higher, and the bandwidth is about 65% broader than the conventional dual toroidal phase shifter. Beyond this optimum value, increasing  $s/a$  decreases both nonreciprocity and bandwidth.

Fig. 6 shows the effects of the thickness of the high dielectric layer (the dielectric thickness =  $2d_d$ ) on non-

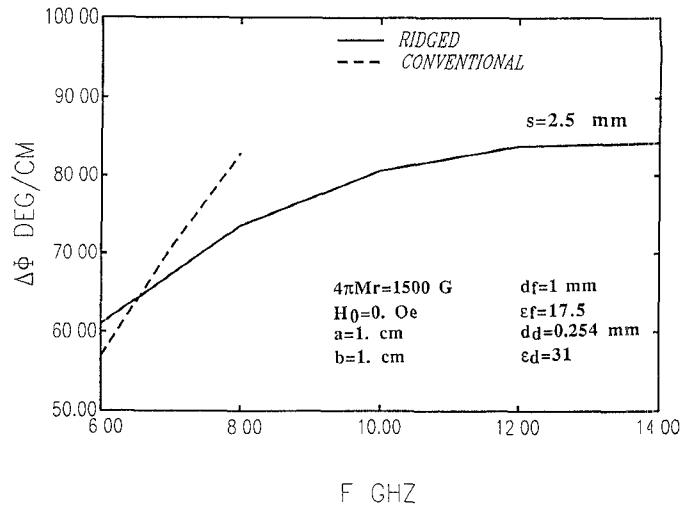


Fig. 3. Differential phase of a ridged waveguide versus frequency.

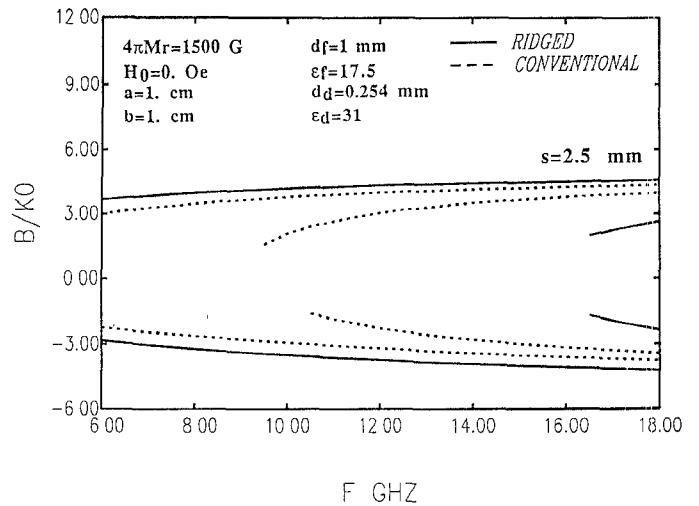


Fig. 4. Different modes of ridged and conventional waveguide phase shifters.

reciprocity and bandwidth. Fig. 6(a) shows that for a given ridge separation  $s$ , the differential phase increases with increasing thickness of the dielectric layer (or the ridge width) until an optimum thickness is reached; then the differential phase starts to decrease with increasing thickness of the dielectric. This has been found to occur when the thickness of the high dielectric constant material is approximately half of the ridge separation (i.e.,  $s \sim 4d_d$ ), providing that  $s/a$  is relatively small ( $s/a < 0.5$ ). As mentioned above, the high dielectric layer increases the field ellipticity and enhances performance. However, as the thickness of this high dielectric material increases, more fields are concentrated in the dielectric which is reciprocal. As a result, the nonreciprocity starts to decrease beyond an optimum thickness of the dielectric (or the ridge width). Increasing the thickness of the high dielectric material usually results in early overmoding which decreases the bandwidth. Fig. 6(b) shows the upper frequency limit of a single mode operation, normalized to that when  $d_d = 0$  (i.e., the case of a slot line between two toroids without

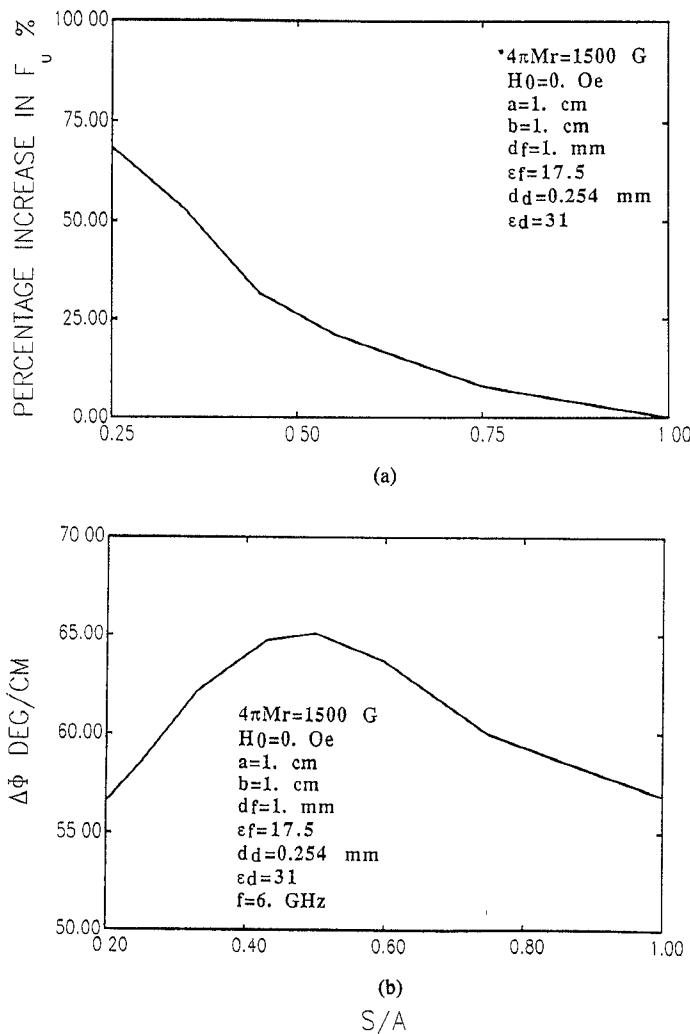


Fig. 5. Effect of the ridge separation on (a) bandwidth and (b) differential phase of ridged waveguide phase shifter.

a high dielectric material which has  $f_{u0} = 19$  GHz). Note that the improvement in bandwidth is still significant compared to conventional waveguide phase shifters (about twice the bandwidth of conventional phase shifters for the same nonreciprocity).

Finally, the effects of increasing the permittivity of the dielectric material have been investigated and are presented in Fig. 7. The nonreciprocity increases almost linearly versus the dielectric constant and reaches values in excess of  $100^\circ/\text{cm}$ . Note that, in general, the upper frequency limit is inversely proportional to the  $\sqrt{\epsilon_r}$ , therefore, the bandwidth-nonreciprocity product is proportional to  $\sqrt{\epsilon_r}$ . Usually, as the dielectric constant increases, so does the loss tangent of the material which increases the phase shifter losses. The nonreciprocity can also be enhanced by increasing the toroidal wall thickness. The thickness of ferrite is usually limited by the dimensions of the phase shifter. Other aspect ratios and parameters of the ridged waveguide phase shifter including the depth of the trough  $b$  have been studied, and their effects have been found to be negligible compared to those discussed above.

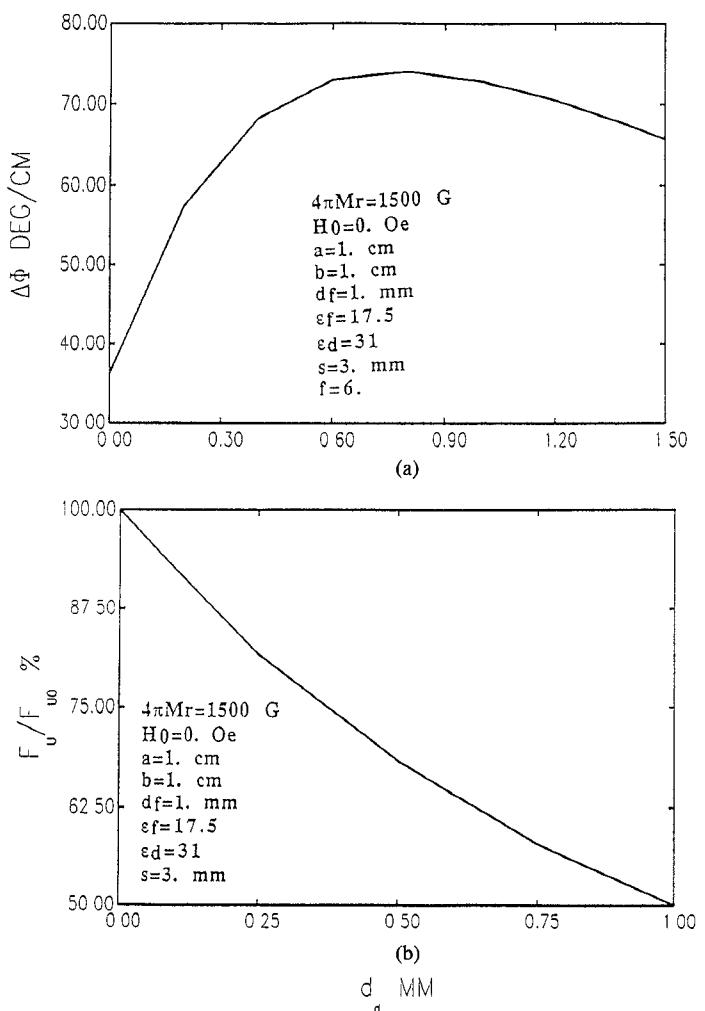


Fig. 6. Effect of the dielectric layer thickness (or ridge width) on (a) the differential phase and (b) bandwidth of ridged waveguide phase shifter.

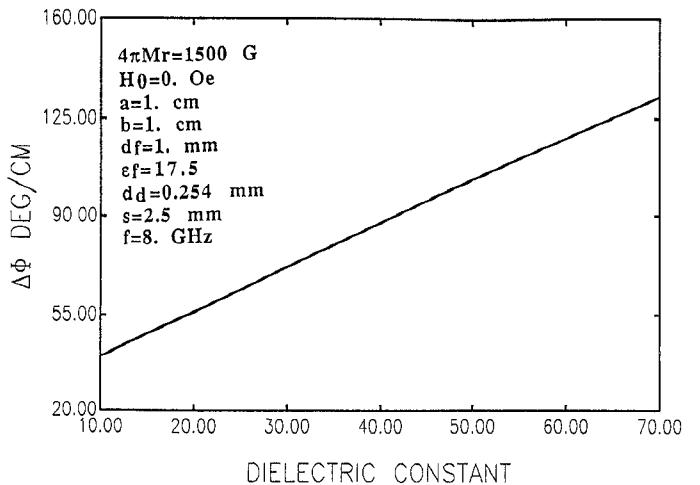


Fig. 7. Effect of the dielectric constant on the differential phase of ridged waveguide phase shifter.

#### IV. CONCLUSION

A double ridged waveguide that employs the concepts discussed in Section I to maximize both performance and bandwidth is presented. Bandwidths twice that of conven-

tional waveguide phase shifters with less variation in the differential phase versus frequency have been predicted for the new phase shifter. Optimum ridge width and separation at which the nonreciprocal performance is maximized have been found. Estimates of these optimum dimensions are  $s = a/2$  and  $d_d = s/4$ . The analysis shows that the bandwidth can be further increased at the expense of nonreciprocity using suboptimal design rules.

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